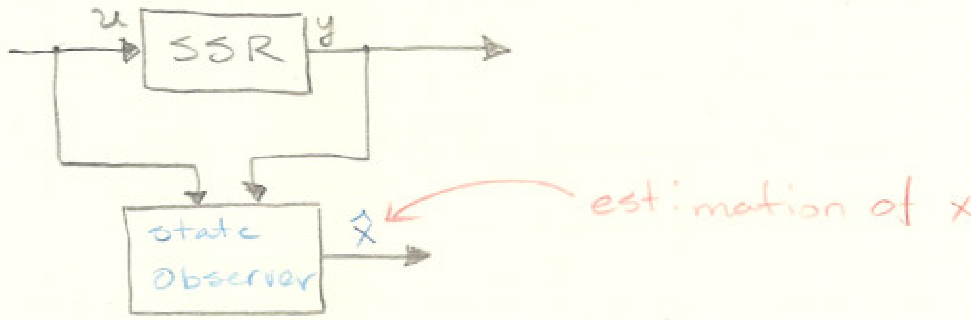
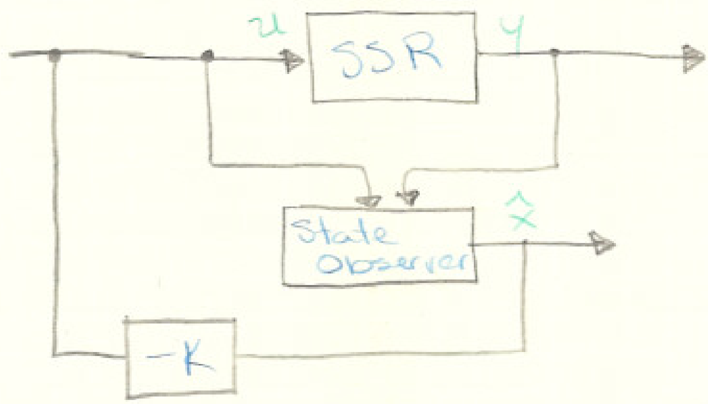
OBSERVER DESIGN.



if the state observer is well designed $\hat{x} \to x$. $t \to \infty$



State observer.

$$\boxed{\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(y-\hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned}}$$

Estimation error. let $\tilde{x} = x - \hat{x}$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu} - L(y-\hat{y})$$

$$\dot{\tilde{x}} = A\tilde{x} - L(y-\hat{y}) = A\tilde{x} - L(Cx - \hat{x})$$

$$\dot{\tilde{x}} = (A - LC)\tilde{x} \Longrightarrow \tilde{x}(t) = e^{(A-LC)t}\,\tilde{x}(0)$$

if L is designed such that the eigen values of $(A-LC)$ are all with negative real parts then $\tilde{x}(t)$ is bounded and $\tilde{x}(t) \underset{t\to\infty}{\to} 0 = \forall \tilde{x}(0)$

we can design L such that $(A-LC)$ is stable if and only if the system is completely observable.

OBSERVABILITY CONDITION.

iff $\text{rank}(O) = n$

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

EX:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$O^T O = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix} \qquad \det(O^T O) = 10 \neq 0$$

$\therefore$ $\text{rank}(O^T O) = 3$ $\therefore$ completely state observable.

EX:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Design a state observer such that the desired eigenvalues are $[-2 \;\; -4]^T$

check observability.

$$O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \det(O) = 1 \neq 0$$

$\therefore$ rank$(O) = 2 = n$ $\therefore$ completely state controllable

$$A - LC = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -L_1 & 1 \\ -1-L_2 & -2 \end{bmatrix}$$

$$\lambda I - (A - LC) = \begin{bmatrix} \lambda + L_1 & -1 \\ 1 + L_2 & \lambda + 2 \end{bmatrix}$$

det ...

$$\left.\begin{array}{l} (\lambda + L_1)(\lambda + 2) + (1 + L_2) = 0 \\ \\ \text{match with} \\ \\ (\lambda + 2)(\lambda + 4) = 0. \end{array}\right\} \text{solve for } L_1, L_2$$